Capilano University Physics Labs 110/111/114/115 (Vern Moen, Jan 05, 2011)
Appendix A: Uncertainty, both Random and Systematic.

Uncertainties Generally:

What is uncertainty, exactly? Is it mistakes that have been made? Is it imprecision in measurement? Is it doubt? Is it an unavoidable consequence of the probabilistic nature of the measurement process? Is it an unavoidable consequence of the probabilistic nature of the quantities we measure?

It is all of the above, and more, in the Sciences.

As a young professional in training a lot of your effort and thought will go into uncertainties. The better prepared you are on the subject, the more rewarding the physics labs will be. The labs provide an excellent context to practice your new found knowledge of this complex topic.

You will be pleased to that uncertainties can be handled reasonably well without a lot of pain, and even with confidence. All you need to master are a few simple fundamentals, and an approximation or two, and you will be well on your way. With experience in the lab your knowledge of the subject will become more robust.

Lets get started on some of the simpler fundamentals:

Systematic Uncertainties:

Suppose you used a meter stick to measure a bunch of lengths. You failed to notice that the stick had been cut off at the 1cm mark instead of at 0 cm. All your measurements are subject to the same effect of this 'mistake': They are all 1cm longer than they should be. This is one form of **Systematic Uncertainty.** It is not uncommon for this to be called all sorts of other things (such as systematic error, mistake, etc) but it doesn't really matter what you call it. All you have to do is subtract 1cm from all the values and the problem goes away. That is the good thing about this type of systematic uncertainty: They can almost always be made to go away, but only if you **notice them** in time to measure them and correct for them.

Suppose you used a meter stick to measure a bunch of lengths. You failed to notice that it was actually scaled in inches, and not centimeters. All your measurements are subject to the same effect of this 'mistake': They are all (1 centimeter/1 inch) smaller than they should be. This is another form of **Systematic Uncertainty.** It might also be called a scaling error, but again it doesn't really matter what you call it, and it can be corrected for if you notice it.

The good news is that, if you make a mistake, you can call it a systematic uncertainty, which in general is anything that causes all the results to be shifted or scaled in some consistent way. The KEY and ESSENTIAL point here is this: If you find the systematic uncertainty before it is too late, then you must measure it and correct your results for it, thus making it go away. In fact it is no longer referred to as an uncertainty or error. It need not even be referred to, because it has been 'corrected out'. If you don't find the systematic error then your results may prove to be 'strange' in some way. That is when you refer to a 'systematic uncertainty' to explain away the mystery. Systematic uncertainties are never a happy event for an experimentalist. They almost always mean that more work has to be done.

Uncertainties (the normal random kind :)

It is strange when you first read it, but science is about quantifying things, and especially uncertainty. We put a number on uncertainty, and we are reasonably certain about that number. **Quantifying the uncertainty** of anything you measure or calculate is the most important job in experimental science.

A simplifying idea to start with:
You all have built-in circuitry to assess risk. It is fundamental to survival. You all have built-in circuitry that provides fault-tolerance. The concept of 'close enough' is innate to everyone. The careful balance between 'risk and reward' is continually being processed by you, albeit subconsciously.

We need to 'tap in' to that 'risk and reward' circuit in your brain for you to get an intuitive handle of uncertainties, and to give you a reasonable, simple tool to estimate uncertainties.

The first step is to realize that when we measure something we are looking for two numbers, and not one. We typically 'measure' one. We take the ruler, or the scale, and use it to write down one value. A little definition here: The value we normally write down is called the 'best estimate'. It is our current best guess for what the value is. But in fact we presume up front that we don't know the value EXACTLY. Lets say we want to know your weight. We won't ever know it EXACTLY. It changes even as we think about it. (Think about it... you are respiring, perspiring, and consuming some energy thinking about it). But we are safe in presuming that you are close to the value a scale would give us. We write down that value, and everyone reading it knows that we are not EXACTLY that weight. We are APPROXIMATELY that weight. That means that we are uncertain about the exact weight, but reasonably confident that we are around that weight.

As noted above, in science we are all about quantifying things. We are going to improve on that 'best estimate' but giving an estimate of how far away our best estimate is likely to be from the actual value. That is a mouthful. We give that estimate in the form of an uncertainty. We say that the distance from our best estimate to the actual value is probably less than the uncertainty.

"I weighed myself at 165 lbs last week on my bathroom scale. Its not a very good scale, but I am guessing that its within 5 lbs of my actual weight."

Brilliant. We not only have a best estimate of the weight, but also an estimate of the uncertainty. We have a pretty clear picture here.

"I weighed myself at 165.0 lbs last week on my new digital bathroom scale. Its supposed to be pretty accurate, so I am guessing that its within 1lb of my actual weight."

Our confidence (the antithesis of uncertainty) is dependant on the circumstances as noted above.

To be safe, we could have said:

"I weighed myself at 165.0 lbs last week on my new digital bathroom scale. Its supposed to be pretty accurate, so I am guessing that its within 1000 lbs of my actual weight."

Now clearly this statement is true. It is almost inconceivable that it is wrong, so in the sense that it is 'safe', it is a better statement. At the same time, it is quite useless to tell me that your actual weight less than 1000 lbs away from 195 lbs. It tells me even less than I could guess myself without any information at all.

We are expecting a statement much more like the first and second one, in which the person has given an estimate of the uncertainty that is not too safe, but not too unsafe either:

“I weighed myself at 165 lbs last week on my bathroom scale. Its not a very good scale, but I am guessing that its within 0.01 lbs of my actual weight.”

Guess again! That is way to unsafe for an uncertainty! The statement is almost certainly wrong!

So from these observations we come up with a simple scheme (called an algorithm) for making a useful estimate of the uncertainty of any best estimate. You must practice this to get fast at and confident at it:

Simple algorithm for estimating uncertainties:

Estimate an uncertainty as some power of 10 that is clearly ridiculously safe:
“I weighed 195lbs on my bathroom scale, which is within 10000 lbs of my actual weight”.
Now reduce this ridiculously safe estimate by a factor of 10:
“I weighed 195lbs on my bathroom scale, which is within 1000 lbs of my actual weight”.
Repeat:
“I weighed 195lbs on my bathroom scale, which is within 100 lbs of my actual weight”.
Repeat:
“I weighed 195lbs on my bathroom scale, which is within 10 lbs of my actual weight”.
Repeat:
“I weighed 195lbs on my bathroom scale, which is within 1 lbs of my actual weight”.
Until: you are feeling unsafe about the range, given all the circumstances you want to consider.
Now test the following values: 2 time this last value, 5 times this last value, and 10 times this last value, until you settle on a value that feels neither particularly safe, nor particularly unsafe.
Bingo! You have the value you are looking for: A gamble that is about an even money bet; a range that you are *uncertain* if it is safe or unsafe.
“I weighed 195lbs on my bathroom scale, which is within 5 lbs of my actual weight”.
The good news about this guessing game: Quite reliably you will go from 'safe' to 'unsafe' very quickly.

Symbol Definition: Δ
Here we need a symbol definition. If I measure a length 'L', and I claim that there is an uncertainty in this length, then the symbol for 'the uncertainty of' is 'Δ', and therefore the symbolic representation of the uncertainty in the length is ΔL. This 'short-hand' version for the uncertainty in the length is quite a time saver, and also is required for writing equations.

Symbol Definition: Best Estimate
If a am measuring a length, L, then the best estimate of L is symbolized $\bar{L}$ The 'over bar' is sometimes omitted in writing best estimates for convenience. However clarity is sacrificed.

The standard representation for every value you measure or determine (measure or calculate) is
$L=\bar{L}\pm\Delta L$ where the symbols are as defined above.

Calculated Values:

It could get ugly. But in this lab it will not. The rules we establish for calculating uncertainties are few and simple. The only imperative is that you calculate them separately from the best estimates, using the appropriate equations, and the rest will be uncomplicated:

Rule One: Uncertainties are always quoted as positive, even when they are calculated as negative.

Suppose we **add** two values: $f=y+x$
then the first rule is obvious: $\bar{f}=\bar{y}+\bar{x}$
This next seems obvious:
$\Delta f=\Delta y+\Delta x$
and finally we write $f=\bar{f}\pm\Delta f$ where we substitute in the values we have calculated above.
The last equation reads: "The value of **f** is the best estimate of **f** plus or minus the uncertainty of **f** "

Suppose we **subtract** two values: $f=y-x$
then the first rule is obvious: $\bar{f}=\bar{y}-\bar{x}$
This next one is not obvious, so you will have to learn it:
$\Delta f=\Delta y+\Delta x$ (the exact same as for subtraction, note)
and finally we write $f=\bar{f}\pm\Delta f$ where we substitute in the values we have calculated above.

Suppose we **multiply** two values: $A=L*W$
then the first rule is obvious: $\bar{A}=\bar{L}*\bar{W}$
which says that we multiply the best estimates to get the best estimate of the result.
This next one is not obvious, so you will have to learn it:
$\frac{\Delta A}{\bar{A}}=\frac{\Delta L}{\bar{L}}+\frac{\Delta W}{\bar{W}}$ therefore $\Delta A=\bar{A}*(\frac{\Delta L}{\bar{L}}+\frac{\Delta W}{\bar{W}})$
and finally we write $A=\bar{A}\pm\Delta A$ where we substitute in the values we have calculated above.
Suppose we **divide** two values: $A=\frac{L}{W}$
Skipping the obvious (by now), we must learn that: $\Delta A=\bar{A}*(\frac{\Delta L}{\bar{L}}+\frac{\Delta W}{\bar{W}})$
(which is exactly the same as for multiplication).

Suppose we **exponentiate** a value: $f=y^n$ for any real number n (positive or negative)
we must learn that: $\frac{\Delta x}{\bar{x}}=n\frac{\Delta y}{\bar{y}}$
Suppose you take the **logarithm** of a value: $f=\ln(y)$
we must learn that: $\Delta f=\frac{\Delta y}{\bar{y}}$
Suppose a value is an **exponential**: $f=\exp(y)$
we must learn that: $\Delta f=\Delta y*\exp(\bar{y})$

Suppose a value is inside a **sin function**: $f=\sin(y)$
we must learn that: $\Delta f=\Delta y*\cos(\bar{y})$ where **y is converted to radians**

Suppose a value is inside a **cos function**: $f=\cos(y)$
we must learn that: $\Delta f=\Delta y*\sin(\bar{y})$ where **y is converted to radians**

Suppose we have a **general function**: $y=F(x)$
then the first rule is obvious: $y=F(\bar{x})$
This next one is based upon the differential approximation. You do not have to learn it:
$\frac{dy}{dx}=\frac{dF(\bar{x})}{dx}$ which in differential form is: $dy=dx*\frac{dF(\bar{x})}{dx}$
which in finite differences is: $\Delta y=\Delta x*\frac{dF(\bar{x})}{dx}$ If we can find the slope at x, then we multiply this slope by the uncertainty in x, which give us the uncertainty in y.
Finally we write $y=\bar{y}\pm\Delta y$ where we substitute in the values we have calculated above.

So far we haven't used a single number. That is a good thing: It means that there are general, simple rules to master that apply to all cases you will encounter in this lab. But soon enough you will have to work with the numbers. Special rules will apply:

When it comes to the numbers:

Uncertainties that you estimate using the above rule for stuff you measure will be written to 1 significant figure, this significant figure being 1, 2 or 5. These uncertainties will have units. All uncertainties, in general, will have units.

Here I will define an **abbreviation** that will be used throughout the course: 'significant figures' will be written sig fig from now on, representing both the singular and plural forms of the word 'figure'.

If you calculate an uncertainty, then the following rules apply: All uncertainties are calculated to 2 sig fig. All calculated uncertainties are carried to 2 sig fig in subsequent calculations. All uncertainties are quoted to 2 sig fig. This is known in this lab as the **2 Sig Fig Rule**.

If a value is calculated or measured, then presented, it will be presented in **Proper Final Form**:
$y\ =\ \bar{y}\pm\Delta y\ =\ (***.*\pm?.*)*10^{n}\,units$
`(where ? is a digit` between 1 and 9, and * is any digit)
This means the following rules apply:

- the units must be outside the bracket
- no powers of 10 are permitted inside the bracket
- The uncertainty must have exactly 2 significant figures
- The best estimate must have the same number of decimal places as the uncertainty

The percent uncertainty:

There is, by convention, a 'percent uncertainty' that is sometimes used in discussions and conclusions of reports, and sometimes in casual conversation amongst experimentalists. There is no conventional symbol for the percent uncertainty.

However, if 'x' is the value that is being looked at, then its percent uncertainty is unambiguously calculated by the equation:

The 'percent uncertainty of x' $= \frac{\Delta x}{x} * 100\%$ (Note that 100% = 1)

Use percent uncertainty with caution, and only if required.

For discussions on **Comparison of two values**, **Agreement of two values**, and '**To what degree..**", please refer to the existing Appendix A.

Comparison of two values (addendum):

Note in your readings, however, that there is no universally defined symbol or abbreviation for 'the percent difference'. In order to avoid confusion, you are advised to write the left hand side of the equation as below:

The 'percent difference' = $\frac{A-B}{A}$ if you are comparing two values 'A' and 'B'. It is essential to remember that in this case we must accompany this calculation with a statement: "B is __% greater than A". This simple statement tells us we decided to divide by A. Of course, we would write: "B is __% less than A" if this were the case. If we used the equation:

The 'percent difference' = $\frac{A-B}{B}$ then we would have to write: "A is __% greater than B" because we were dividing by B, and therefore comparing to B in this case.

Agreement of two values (addendum) :

It is important to note that it is a much stronger statement to say that your value **does not agree** with an expected value, than it is to say that it **agrees**. You must be particularly cautious in coming to that conclusion. You must be fairly confident in your work and in your uncertainties, then show that the results differ by more than twice the uncertainties (combined). You see, there is better than a 1/3 chance that your result will disagree by more than the uncertainty. You have to go all the way to twice the uncertainties to reduce that chance to 5%. Do you remember the exercise in estimating uncertainties above? You have a fair distance to go before you change 'uncertain' to 'safe', or 'unsafe'.

To what degree..(addendum):

Synonyms for "To determine the degree to which ..":
"To determine how well ..."
"To determine to what extent .."
All choices above presume that there will **not** be agreement as defined above (unless the uncertainties are particularly large and forgiving). There is some underlying reason that the values are not expected to agree. It is presumed that your experimental results may show this.

Further Reading: Examples, Exercises, and Reminders:

I am certain that you will need examples of these rules being applied. Furthermore, there is a definite necessity for you to practice these rules and methods before attending class. In addition to hand-outs in class, please refer to the following link:
http://www.capphysics.ca/, then select Phys 110/114 Labs, then Web Help
or go directly to: http://www.capphysics.ca/PhysLab/Phys114115/Help/index.htm