Physics Lab 110-114, Spring 2011

Exercises for Calculating Uncertainties under Additon and Multiplication

provided by Vern Moen Jan 13, 2011 (edited Sept 21, 2011)

Answers follow (in proper final form, and including the proper context)

Exercise 1:

Suppose I had to know the total weight of a bunch of stuff that was going to be shipped out.

Suppose I know in advance that the stuff will be organized into identical boxes of identical stuff, but I am not sure how many boxes of this stuff there will be. Let N be the number of boxes of stuff, then the uncertainty of N will be ΔN. Let the weight of each box be W, with an uncertainty of ΔW. Let 't' be a subscript we define as 'of the total', then the total weight of the shipment ($W_{t)}$ will be given by:

$W_t = N * W$ and the uncertainty is given by: $\frac{\Delta W_t}{W_t} = \frac{\Delta N}{N} + \frac{\Delta W}{W}$

which we solve for ΔW_t to give:

$$\Delta W_t = W_t * \left(\frac{\Delta N}{N} + \frac{\Delta W}{W}\right)$$

This is great. Suppose the weight of each box was reasonably expected to be within 3kg of 28kg, and the number of boxes was going to be 140, give or take 12 boxes. Using the equations above, what would your best estimate of the total weight of the shipment be, and within what range of the actual weight would your best estimate likely be?

Exercise 2:

The same as in exercise 1, except you didn't know the weight of each box. Instead you only knew the weights of the items in each box: Each box was to contain two small boxes of stuff, and a stapler. One stapler was measured to have a mass of 501.4g, which is likely to be within 2 grams of the mass of all the staplers. The first of the two small boxes will contain approximately 100 pens (each box likely being within 5 of 100), and each pen weighing within 0.2 grams of 7.6 grams. The second of the two small boxes is going to contain bolts. Each box will have 25 bolts in it, counted carefully. The bolts start out with nuts on them. Each bolt and nut weights 45g , ± 2 g,
and a nut by itself weighs 4.7 ± 0.3 g.

Now find W_t and ΔW_t, but using the new information about W (the weight in each box). Start out by symbolizing everything we need to know. Then write down the necessary equations using these symbols. For instance, you might write down that the weight of the small box containing the pens is W_p , the weight of the box containing the bolts is W_b and the weight of a stapler is W_s . The total weight of a box (W) is just the sum of these. But what about its uncertainty, ΔW ?
$\Delta W = \Delta W_p + \Delta W_b + \Delta W_s$, because the values are added. So finding W_t and ΔW_t will be easy when we know W_p, W_b, and W_s .

$W_p = N_p * W_{p\,each}$, where $W_{p\,each}$ is the weight of each pen, and N_p is the number of pens per box.
Therefore $\frac{\Delta W_p}{W_p} = \frac{\Delta N_p}{N_p} + \frac{\Delta W_{p\,each}}{W_{p\,each}}$. Solving for ΔW_p gives $\Delta W_p = W_p * \left(\frac{\Delta N_p}{N_p} + \frac{\Delta W_{p\,each}}{W_{p\,each}}\right)$

Noting that $N_p = 100$, $\Delta N_p = 5$, $W_{p\,each} = 7.6\text{g}$, and $\Delta W_{p\,each} = 0.2\text{g}$

do the substitutions and find W_p and ΔW_p .

Using the same approach, find the mass of a bolt without a nut (assign symbols, write down the correct equations, then do the substitutions until you have the mass of a bolt and its uncertainty. Now note how many bolts there are in a box, and find the mass of the box with the bolts in it. Note that, under the circumstances, the number of bolts could be considered a constant with no uncertainty.

Finally you should be able to calculate W_t and ΔW_t . What remains is to put the result in final form.

Exercise 3: Same as exercise1, but make up different numbers.
Exercise 4: Same as exercise 2, but make up different numbers.

Repeat until you have these examples 'burned in'.

Answers (in proper final form, and including the proper context):

Exercise 1:
The the best estimate of the total weight of the shipment would be: 3920 kg, and this estimate is likely to be within 780 kg of the actual shipment weight. The total weight of the shipment ($W_{t)}$ is
$W_t = (392 \pm 78) * 10$ kg

Exercise 2:
The weight of a box pens (W_p) will be $W_p = (760 \pm 58)$ g
The weight of a bolt without a nut would be $W_b = (40.3 \pm 2.3)$ g
The weight of each shipped box (W) would be W = (1302 ± 62) g
And the total weight would be
$W_t = (182 \pm 24)$ kg

For a different set (and type) of exercises, see the document:
Multiplication/Division worksheet on the website, Physics 110-114 under Web Help,
Vern's help Spring 2010.